(5) A series of light aluminium alloys has been tested with the balance, and it has been found that, whereas the susceptibility of commercial aluminium is increased by alloying with copper and manganese, it is diminished by alloying with cobalt.

(6) In a note on the susceptibility of glass in relation to composition it is shown that the balance could be used to rapidly determine the relative amounts of ferrous iron in different specimens of glass.

(7) Certain specimens of tourmaline have been examined. The green and dark blue opaque varieties have susceptibilities in the direction of the principal crystallographic axis varying from 16 to 20 per cent. less than in a direction at right angles thereto. The susceptibility of rose-coloured tourmaline is very small in comparison.

(8) The paper concludes with a note on the retentivity of rock specimens and its possible influence upon magnetic disturbances in magnetic survey work.

On the Variations of Wave-length of the Oscillations generated by Three-electrode Thermionic Tubes due to Changes in Filament Current, Plate Voltage, Grid Voltage, or Coupling.

By W. H. Eccles, D.Sc., and J. H. Vincent, M.A., D.Sc.

(Communicated by Prof. W. H. Bragg, C.B.E., F.R.S. Received November 20, 1919.)

It is well known that a circuit containing a condenser and inductance coil can be maintained in oscillation at a frequency near its natural frequency by aid of a three-electrode thermionic vacuum-tube and suitably connected batteries. It is also known that although the frequency of the oscillations depends mainly upon the magnitudes of the inductance and the electrical capacity it is also affected by the resistance in the oscillatory circuit, by the voltages of the various batteries in use, by the temperature of the filament supplying the electrons, by other properties of the vacuum tube, and by the coupling between portions of the circuit associated with the grid and the anode.*

It is again well known that when one such vibrating circuit is caused to induce current in an independent oscillating circuit the induced current beats

* See, for example, a paper by W. H. Eccles, "Vector Diagrams of some Oscillatory Circuits used with Thermionic Tubes," 'Proceedings of the Physical Society of London,' vol. 31, Part 3, April 15, 1919.

with the local oscillatory current, the beat frequency being equal to the difference between the frequencies of the induced and the local oscillations. This is applied, for instance, in the so-called auto-heterodyne or endodyne method of reception familiar in wireless telegraphy, where the difference of the high frequencies is arranged to be of acoustic frequency so as to operate the telephone receiver connected with the local oscillatory circuit. In this application the operator varies the pitch of the sound made by the beats by altering the capacity of his tuning condenser. Starting with the variable condenser adjusted to an extreme position such that the local frequency is, say, 10,000 per second lower than the frequency induced by the distant apparatus, a very shrill note is heard in the telephone, and as the local condenser is diminished in capacity, the pitch falls continuously through all the audible octaves till the beats are so slow, say 30 per second, that they cease to form a note.

Continued reduction of the capacity brings a condition in which the beats are below the lower limit of audition till at last the local circuit is "in tune with" the distant oscillator and there are no beats. Further reduction of the local capacity brings the local frequency higher than that of the distant apparatus, and when this difference reaches about 30 per second the beats become audible again; continued gradual reduction of the capacity now causes the pitch of the note to ascend the scale, till at length it disappears beyond the upper limit of audition.

In the following pages, when "the region of silence" is spoken of, the adjustments of the apparatus implied are those in which the relative frequency of the two co-operating maintained oscillations is less than about 30 vibrations per second. The "region of silence" is not actually utilised in the experiments; we found the most advantageous method in our work of tracing the small changes of frequency brought about in either circuit, was to adjust the apparatus so as to obtain a note of convenient pitch and to observe changes in this pitch.

The object of the present investigation was to study experimentally the effects of altering each of the chief variables, with a view to finding the conditions most favourable for the production of continuous waves of constant frequency.

Apparatus.

Two oscillating circuits were set up on two work benches placed a few feet apart, and were maintained in oscillation separately by small thermionic tubes of the kind used in the British Army. In addition to the maintained oscillatory circuits, a third circuit was set up on Bench 1. In this a crystal detector and direct-current micro-voltmeter were connected. This third

circuit will be called the crystal circuit. When not being used its inductive coil was removed.

Bench 2 circuit differs from that on Bench 1, in that the plate battery which connects with the oscillator is broken to insert a telephone. The plate battery and telephone are shunted by a condenser (C.T., fig. 1). The circuits on Benches 1 and 2 cooperate to affect the telephone, because of the presence of the linking coils shown at the top of fig. 1.

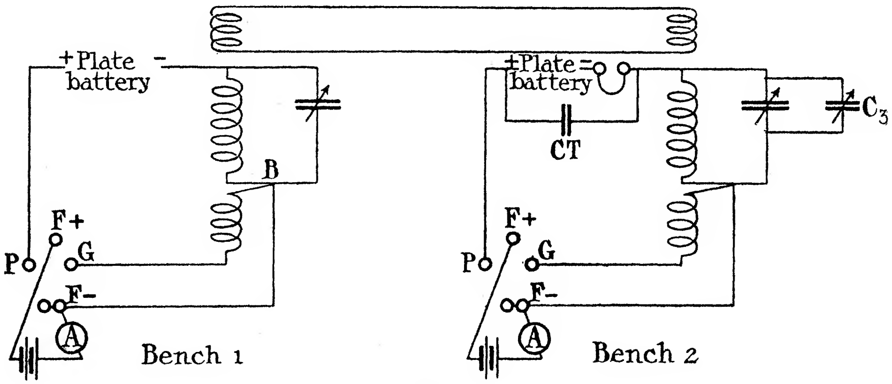

Fig. 1.

The electrical capacity in Bench 2 circuit is made up of two condensers, one of relatively large and fixed capacity, and the other, C_3, of smaller capacity joined in parallel with the first. The small condenser could in some experiments be used for adjusting the frequency of Bench 2 circuit throughout a series of changes in the Bench 1 circuit, so as to keep the circuits at a fixed difference of frequency. In other cases, however, when the changes in frequency in the Bench 1 circuit were large, the larger condenser was used. Both condensers were calibrated.

Method of Observation.

The tuning was done by beats. The capacity of the condenser in the Bench 2 oscillator was first adjusted until a musical note was heard in the telephone. There are two possible adjustments for this, one on either side of the position of silence. If necessary, the capacity in Bench 2 was changed so that the setting of the condenser was on that side of the position of silence in which increasing the capacity raises the pitch of the note. The note was next tuned to unison with a tuning-fork heard simultaneously. This fork has a frequency of 256 complete vibrations a second. There are now two closely adjacent values of the capacity in which one beat per second is heard. That setting of the condenser in which the capacity is the larger of

these two values gives a frequency to the Bench 2 circuit, which is less than that of the Bench 1 circuit by 257 complete vibrations a second. The accurate adjustment of the beat to one per second was accomplished by aid of a metronome.

Effect of Change in Filament Current on Wave-length.

The wave-length of an oscillator is altered slightly by changing the filament current. As the filament current is raised from the lowest value for which a note is heard in the telephone, the wave-length rises to a maximum, and for further increase in the filament current the wave-length decreases. This is shown in fig. 2, which gives the results of one of many sets of observations all agreeing in their general character. The actual position of the curve on the axis of filament current, and its shape, will depend on the tube itself as well as upon the circuit in which it operates. For these measurements the Bench 2 filament current was kept constant, while that in the Bench 1 tube was altered. The wave-length in this particular apparatus reaches a maximum when the filament current is about 0·77 ampère. The actual values of the changes in wave-length can be readily computed from the known constants of the Bench 2 circuit. In this case the total change of wave-length from the lowest value to the maximum is about 0·3 per cent., that is about 9 metres, since the waves were 3000 metres long.

If now the filament current in the tube of Bench 1 oscillator be kept constant, and that in the Bench 2 oscillator varied, then the compensatory changes in the Bench 2 condenser will be of opposite sign. The tendency towards increase in wave-length due to increase in the filament current has to be compensated by a decrease in capacity to keep the wave-length unaltered. We should thus expect the condenser settings and filament current curve to be somewhat of the same shape as that of fig. 2, but upside down. This was, in fact, the case, as is shown in fig. 3.

The generation of oscillations of maximum wave-length by a maintained circuit of the type shown in fig. 1 is not dependent on the exposure of the tube to the air of the laboratory. Similar curves are obtained whether the valves are covered by a bell-jar or not. Nor does this effect depend for its occurrence on the plate voltage having any particular value. One may alter the plate volts from 70 to 125 without materially changing the shape of the curve, and, when this is done, the maxima occur at values of filament current which are not far apart. It is to be noted that the actual values of the condenser settings at the maxima are not the same with different plate voltages. This is discussed below. The curves shown in fig. 4, in which the plate voltages are varied, are thus to be regarded as set down arbitrarily in

respect to the condenser readings, the datum line varying from curve to curve.

Thus the wave-length emitted by an oscillation apparatus is a function of

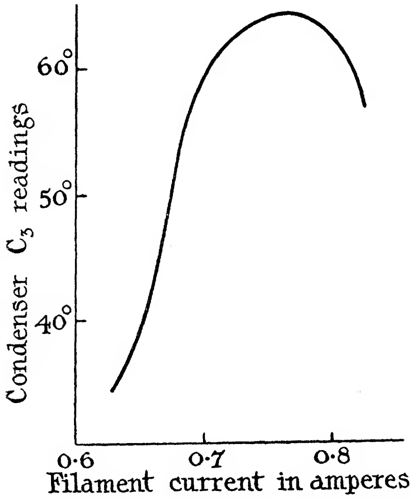

Fig. 2.

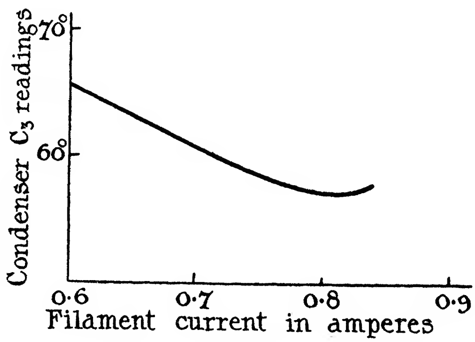

Fig. 3.

the filament current; it is a maximum for a definite current which is not greatly changed by altering the plate voltage. This has been found to occur

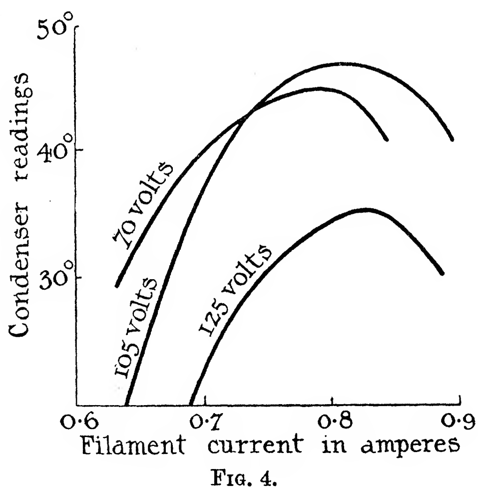

Fig. 4.

in different circuits with different values of inductance and capacity. In this description of the effect of filament current on wave-length, it has been assumed that the coupling between the grid coil and oscillator coil was neither closer than is necessary to maintain the oscillations while the

filament current varied from 0·65 to 0·85 ampère, nor so loose as to make the oscillations too feeble for telephone tuning by beats. If these restrictions are removed, the above statements are still true, but the whole of the curves of the type of fig. 2 may not be accessible for experiment. This will be dealt with further in the last section.

The method of utilising the existence of this phenomenon to obtain constancy of wave-length, in laboratory work of the kind to be described later in this paper, is now clear. In such cases two oscillatory circuits are set up, and one (say the Bench 2 circuit) is used as a delicate wave-meter to study small differences in the wave-length of the oscillations of the other. The Bench 1 circuit has its constants fixed, including a temporarily stationary value for its filament current. The Bench 2 circuit is now used to draw a curve for its own tube like fig. 3. This fixes the filament current to be used in the Bench 2 circuit. In this connection it may be worth noting that, when this current has been determined, no change should be made afterwards in the Bench 2 circuit except the small adjustments of its condenser necessary for tuning. The tube maintaining the oscillations in the Bench 1 circuit is now studied, and a curve like fig. 2 drawn.

We have now determined both filament currents so as to give a constant wave-length in their respective circuits in spite of the unavoidable small variations of filament current. When this is done, the beats heard due to the interference of the notes of the telephone and tuning-fork are, under good conditions, very distinct and steady. That is to say, the frequencies of the two sets do not separate as much as 1 part in 100,000 during the lapse of several minutes. Difficulty is sometimes experienced in getting satisfactory settings of the Bench 2 condenser, but this is usually due to the fact that the telephone note is not pure. And, again, a series of readings may be spoilt by the observer unwittingly setting the condenser so as to allow a note present in the sound from the telephone, but not the fundamental, to masquerade as the fundamental. This is apt to occur in cases when the fundamental is weak or when the fundamental due to any cause has fallen into the region of silence. When the filament currents are adjusted as above the beats are easy to hear, and steady enough for the observer to feel confident of the condenser readings. If the fundamental is inaudible, the beats which can still be heard are spurious, and can be distinguished by their unsteadiness and faintness.

It may happen that on drawing the curve for the change in wave-length with filament current the maximum is not in the range of permissible filament current. It has been found that this is due to the coupling between

the grid coil and main oscillator coil being unnecessarily close. On separating the coils the curve can be made to assume its typical form (fig. 2). If the separation is pushed too far, however, the oscillations may be too feeble before the maximum wave-length is attained.

Applications of the Adjustments Described Above.

Effect of Change of Plate Voltage on Wave-length.

This was investigated with the apparatus already described. The increase in capacity in the Bench 2 circuit necessary to keep it in tune, as the plate voltage was increased in the Bench 1 circuit, is shown in fig. 5, which gives the combined results of two sets of observations as a smoothed curve. Other observations gave the curve more nearly as a straight line, but this subject was not studied sufficiently for us to be sure of the exact shape of the curve.

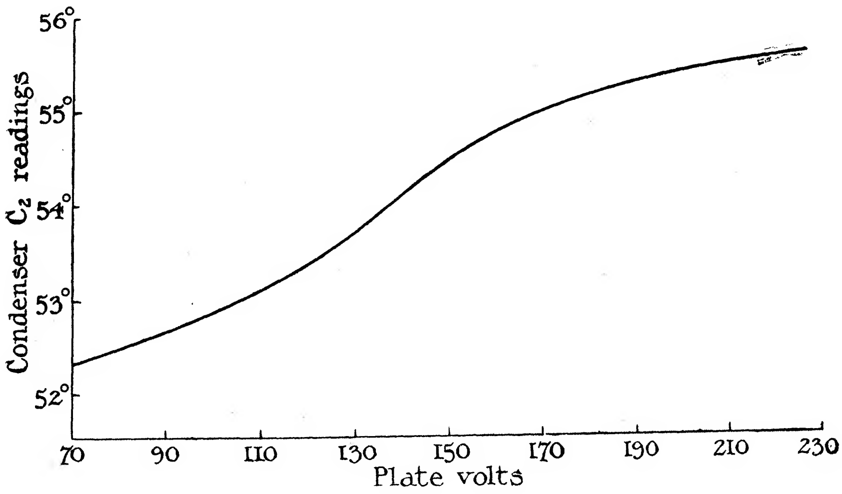

Fig. 5.

The wave-length always increases with increase of plate voltage and a determination of the approximate magnitude of the effect was made. When the wave-length was about 3000 metres and the plate voltage of the tube in the Bench 1 circuit was 130, raising the voltage by 10 increases the wave-length by 6 metres, approximately.

It is worth while considering if this effect is capable of explaining any appreciable part of the change in wave-length with variation of filament current. The mean difference of potential between the plate and the filament (see fig. 1) depends to a small extent on the current in the filament. When this is zero the difference of potential is equal to the plate voltage; but when a current is flowing in the filament in the direction indicated in fig. 1, the

mean potential of the filament is raised, and thus the effective mean plate voltage is decreased by approximately half the fall of potential along the filament. So that when the filament current increases, this effect on the plate voltage decreases the wave-length emitted. But for this curves like fig. 2 would be steeper on the ascending side than they are.

In order to test this point a set of readings of current and voltage on the filament terminals was taken, and the lowest curve on fig. 4 redrawn, with and without allowing for the influence of current on plate voltage. The result is that the effect is not of sufficient magnitude to make any noteworthy alteration in the curves. If the points for 0·65 ampères are taken as coincident, the correction at the maximum only amounts to one-tenth of 1 degree of the condenser in fig. 4.

The immediately applicable lesson taught by these experiments on plate voltage is that plate batteries cannot with impunity be changed or charged during a set of experiments. If a long investigation is in progress care must be taken to keep the plate battery up to a definite voltage, but for work lasting a few hours the small changes in the electromotive force of a good plate battery due to its discharging will not be likely to give any marked change in wave-length.

Effect of Change in Grid Voltage on Wave-Length.

The battery of cells was introduced between B and the grid coil in the Bench 1 circuit (fig. 1) without any kind of potential divider or other mechanical device for altering the applied electromotive force. This alteration was brought about by connecting the cells up in different ways by wires. This crude method of varying the electromotive force was used because of trouble experienced in preliminary experiments, due to changes in inductance and capacity produced by using potential dividers and commutators. Six small (30 ampère-hour) accumulators were placed in a crate and care was taken that all the cells were in metallic connection for each voltage. In this way the results shown in fig. 6 were obtained. The point on the curve for -4 volts is marked with a note of interrogation as the Bench 2 condenser could not be set to produce audible beats between the fundamental and the fork. When the grid voltage was given numerically greater negative values the oscillations ceased.

It is seen from fig. 6 that from -2 to 4 volts the curve is nearly straight, but for higher values of the electromotive force it bends over slightly towards the axis of volts. For this tube and circuit (250 cm. capacity, 8 millihenries) the increase in wave-length is about 10 metres per volt rise in grid voltage, the wave-length being 2750 metres approximately.

One is warned by these results on grid voltage to beware of the change in wave-length produced by inserting electromotive force in the grid circuit. It

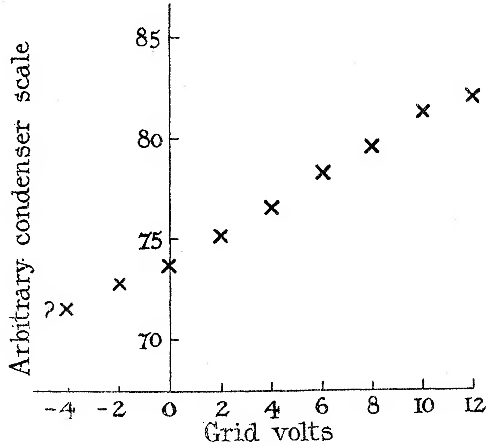

Fig. 6.

is of equal importance to be on one's guard against the effects produced by any change in capacity or inductance that may accompany the alteration in electromotive force.

Effect of Change in Coupling of Grid Coil and Main Oscillator Coil on Wave-length.

In the first section of this paper it has been necessary to refer to the effect of closeness of coupling upon the curve giving the wave-length as a function of the filament current. Some experiments were performed which show that the coupling has a marked effect on the wave-length, other things being fixed.

For a given filament current the closer the coupling the greater the wave-length. For different filament currents the wave-length and filament current curve becomes steeper and its maximum moves towards higher currents as the coupling gets closer. These results are shown graphically in fig. 7.

The five points on the capacity scale for 0·7 ampère filament current are for five different amounts of coupling. Each point represents the added capacity on the Bench 2 circuit necessary for tuning. The grid coil was removed 1 cm. further into the main coil between each pair of points as we pass upwards on the capacity scale. The five curves show the effects on the shape of the curve of the type of fig. 2. We get the typical curve with its maximum in curve *d*. If the coupling is much less the oscillations are too weak for the whole range to be drawn, as in *e*. If the coupling is made closer the curve

gets steeper, the wave-length increases, and the maximum moves to a higher filament current (curve *c*).

In curve *b* the grid coil has been pushed too far into the main oscillator

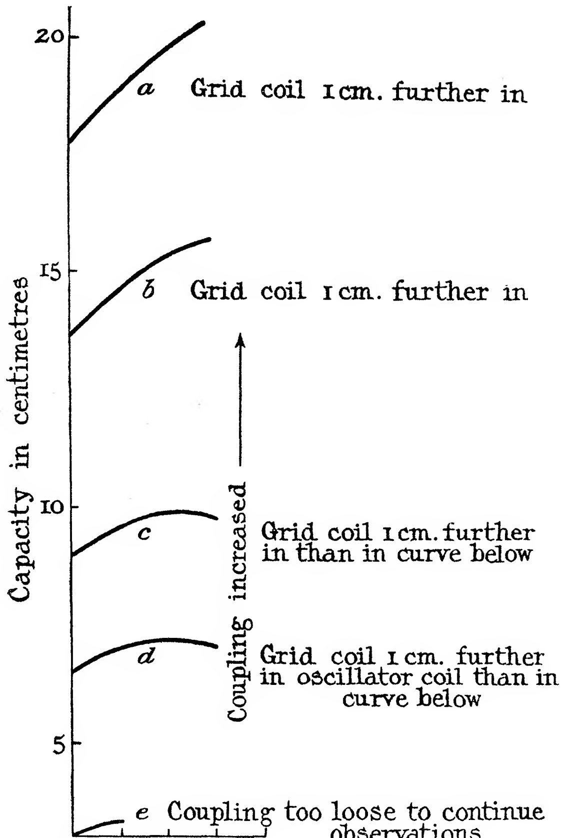

Fig. 7.

coil. We cannot get the maximum without overheating the filament; it has passed out of the diagram to the right. In curve *a* the coupling is so close as to have increased the wave-length materially, and the rising slope of the curve is nearly a straight line. Another set of observations with still closer coupling gave the rising portion of the curve as almost a straight line and still higher up on the diagram.

The variation of coupling gives us a means of putting the wave-length and filament current curve into a position on the axis of filament current in which its maximum is available for use. The coupling can besides be increased so as to give us a filament current and wave-length curve in which

a small change in current produces a large change of the same sign in the wave-length. This property could be applied, for example, to give modulation of wave-length, such as is sought in wireless telephony, or to form the basis of a new method of amplification. In this case the electrical stimulus to be amplified would be introduced into the filament circuit by means of a transformer. The consequent change in wave-length would be the index showing the receipt of the stimulus.

Equal Parallel Cylindrical Conductors in Electrical Problems.

By F. J. W. Whipple, M.A.

(Communicated by Dr. C. Chree, F.R.S. Received June 16, 1919.)

In a recent paper* Dr. Alexander Russell has called attention to the practical importance of the electrostatic problem of determining the distribution of charge and of potential when parallel cylindrical conductors are electrified. Russell points out that the same analysis will serve for the calculation of the current density and the magnetic flux when currents of high frequency pass along the parallel conductors. The solution he gives for the case of equal and opposite charges (or currents) in the two conductors is exact, but in the general case a more elaborate investigation is necessary.

In the present paper the problem is solved by the method of conjugate functions. For the most part, the results are given in forms convenient for numerical calculation. The investigation is confined to the case in which the two cylinders are of the same size; for this case ordinary Jacobian elliptic functions suffice though in general theta-functions are required and the algebra is rather heavy. It is hardly to be supposed that a problem of such a simple character in a favourite field of investigation has not been solved before. It is desirable, however, to have the solution in an accessible form.

2. *Notation.*—The problem to be considered is virtually in two dimensions, the cylinders being represented by circular sections.

Let a be the radius of either circle, $2c$ the distance between the centres, $2f$ the distance between the limiting points Ω and Ω'.

Take the origin midway between the centres and the axis of X along the

* "Electrical Theorems in connection with Parallel Cylindrical Conductors," 'Phys. Soc. Proc.,' vol. 31, p. 111 (1919).